Endogenous entry under Bertrand-Edgeworth and Cournot competition with capacity indivisibility

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Abstract

Strategic market interaction is modelled as a two-stage game where potential entrants choose capacities and active firms compete in prices or quantities. Due to capital indivisibility, the capacity choice is made from a finite grid. In either strategic setting, the equilibrium of the game depends on the size of total demand at a price equal to the minimum average cost. With a sufficiently large market, the long-run competitive price emerges at a subgame-perfect equilibrium of either game. Failing the large market condition, equilibrium outcomes are quite different in the two games, and neither game reproduces the competitive equilibrium.

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1 Introduction[1]

Since Kreps and Scheinkman (1983) (henceforth, KS) there has been active research on Bertrand-Edgeworth competition with endogenous capacity determination. KS have shown that, under the efficient rationing rule, the Cournot outcome emerges at a subgame-perfect equilibrium of a duopolistic two-stage capacity and price game. This result may not hold, though, under

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alternative rationing rules, since a mixed strategy equilibrium of the price subgame can arise on the equilibrium path (Davidson and Deneckere, 1986). More recently, Madden (1998) has established that a uniformly elastic demand curve is sufficient for the Cournot outcome under oligopoly, regardless of the rationing rule. Boccard and Wauthy (2000 and 2004) have shown that the Cournot result extends to oligopoly under KS's assumption on cost and the efficient rationing rule, although this need not be so if, in the short run, the firms can produce above "capacity" at a finite extra-cost.

Throughout this literature the cost of capacity has been represented as a continuous and convex function. It follows that identical firms will choose a positive capacity at an equilibrium of the capacity and price game.[2] Casual observation seems to suggest, though, that there are markets where some potential entrants refrain from entering, though equally competitive as active firms. One natural way to account for such a feature is by introducing nonconvexities in the long-run cost function. In this connection, we take the view that, because of capital indivisibility, the firms are facing a discrete capacity choice set. This results in discontinuities and nonconvexities in the long-run cost function and substantial scale economies. As for the short run, average variable cost is taken as constant up to full capacity utilization, as is customary in the literature on Bertrand-Edgeworth competition. Based on these assumptions, we analyze a capacity and price game under the efficient rationing rule. At a subgame-perfect equilibrium of our capacity and price game, active firms are less than (a large number of) potential entrants. Different types of equilibrium arise depending on parameter values. In the version of the model developed in the main text, the characterization of equilibria is made under the convenient assumption - to be removed in Appendix B - that total demand at a price equal to the minimum average cost is an integer number. It is shown that the equilibrium level of total capacity equals the competitive one - namely, the quantity demanded at a price equal to the minimum average cost -, with pricing behavior on the equilibrium path depending on how large is the market at the competitive equilibrium: with a market large enough compared to the firm minimum efficient scale, the competitive price - the minimum average cost - is charged, otherwise the price subgame has a mixed strategy equilibrium.

The paper also analyzes a capacity and quantity game among Cournot competitors. Similar to price competition, competition among Cournot quantity setters is found to exactly reproduce the competitive equilibrium so long as total demand at a price equal to the minimum average cost is suf-

[2] See below, p. 13.

ficiently large; if not, the firms produce below capacity on the equilibrium path and the equilibrium level of total capacity may exceed, even significantly, the competitive one.

The above results shed further light on the relationship between the outcomes of price competition, Cournot quantity competition, and the competitive equilibrium. It is a well-established property of price-game equilibrium under given capacities that, with the average variable cost constant up to capacity, the (short-run) market-clearing price emerges if each firm capacity is sufficiently small compared to industry capacity (see, for example, Vives, 1986). This result is now extended to the long run: price competition with endogenous capacity determination will *exactly* yield the long-run competitive price so long as, at the long-run competitive equilibrium, the market size is sufficiently large compared to the firm minimum efficient size. The analogous result obtained for the capacity and quantity game is in contrast with Novshek's (1980) model of Cournot competition with entry: in that model, which is based on a smooth U-shaped average cost curve, the equilibrium price in the entry and quantity game tends *asymptotically* to the minimum average cost as the average cost minimizing output decreases relative to total demand.

Finally, our model shows how price competition need not yield the Cournot outcome, even under the efficient rationing rule: if the market is not sufficiently large at the long-run competitive equilibrium, then on the equilibrium path of the capacity and price game active firms play a mixed-strategy equilibrium of the price subgame. This possibility arises because of the discontinuities in the long-run cost function: if the cost function where everywhere continuous and convex, then, at any capacity configuration involving a mixed strategy equilibrium for the price subgame, it would pay the largest firm to reduce its capacity, which would reduce cost without reducing equilibrium revenue.

The paper is organized as follows. The basic assumptions of the model are laid down in Section 2. Section 3 analyzes the capacity and price game, providing full equilibrium characterization according to parameter values. Section 4 similarly analyzes a capacity and quantity game among Cournot competitors. Section 5 discusses the role of capacity indivisibility, also showing how Bertrand-Edgeworth competition would always yield the Cournot outcome under a (weakly) convex cost function. Section 6 briefly concludes. Proofs of some of the propositions in the text are located in Appendix A. Appendix B generalizes determination of equilibria of our capacity and market games when, as is generally the case, the quantity demanded at the minimum average cost is not an integer number. Appendix C deals with a simultaneous capacity and quantity game among Cournot competitors.

2 The model and basic assumptions

We consider the market of a homogeneous product. $D(p)$ and $P(Q)$ denote the demand and the inverse demand function, respectively, p being the price and Q the total quantity. We assume a linear demand curve, $P(Q) = a - bQ$ for $Q \leq a/b$, where $a, b > 0$. At stage 1 there is a set $\mathcal{Z} = \{1, ..., z\}$ of identical potential entrants that make capacity decisions, while active firms compete in the output market at stage 2. To be active, firm i must install some positive capacity $\overline{q}_i$ at stage 1. Capacity is chosen from a finite grid, due to indivisibility of capital and finiteness of available technologies. Let $\mathcal{F}_+ = \{f\}$ with $f = 0, 1, 2, ...$, and $\mathbb{R}_+$ be the set of nonnegative reals. To keep the analysis most simple, availability of a single technology (α) is assumed throughout, except that a clue will also be given as to how the analysis could be generalized under a plurality of technologies. The capacity choice set faced by each firm is taken to be $\mathcal{C}^{(\alpha)} = \overline{\alpha}\mathcal{F}_+$, where $\overline{\alpha} \in \mathbb{R}_+$. The cost of capacity per unit of output is constant at r under full capacity utilization. For notational convenience we let $\overline{\alpha} = 1$, that is, we set equal to 1 the minimum positive capacity that is technically feasible with technology α; consequently, the firm capacity choice set is $\mathcal{F}_+$.

Given $\overline{q}_i$, firm i's cost is thus $c(q_i) = r\overline{q}_i$ for $q_i \leq \overline{q}_i$ (we set equal to 0 the (constant) unit variable cost), while no output can be produced above capacity (equivalently, one can assume $c(q_i) = \infty$ for $q_i > \overline{q}_i$). Clearly the long-run cost function, $C(q_i)$ - showing the minimum cost at any output - is $C(q_i) = r\overline{q}_i$, with $\overline{q}_i = [q_i, q_i + 1) \cap \mathcal{F}_+$. Thus the $C(q_i)$ curve is horizontal at any $q_i \in (f, f+1]$, while jumping up by r units at any f. Clearly $C(q_i)$ is not everywhere (weakly) convex.[3] The long-run average cost curve, $AC(q_i) = r\frac{\overline{q}_i}{q_i}$, slopes downwards for $q_i \in (f,\ f+1]$ (scale economies due to capacity indivisibility), equals r for any $q_i \in \mathcal{F}_+$ (constant returns at full capacity), and jumps up when q_i increases slightly above f (the jump converging to zero as f becomes larger and larger). Clearly, $\min \arg\min_{q_i} AC(q_i) = 1$: the lowest average-cost minimizing output is the minimum capacity allowed for by technology.

A (by assumption) deterministic capacity choice is made by each $i \in \mathcal{Z}$ to maximize the expectation of profits $\pi_i = p_i q_i - r\overline{q}_i$. Henceforth, we denote

[3] Consider any convex combination of f and $f + 1$, with $f \in \mathcal{F}_+$ and weights k and $1 - k$: producing this output costs $C(kf + (1 - k)(f + 1)) = r(f + 1) = rf + r$, which is greater than $kC(f) + (1 - k)C(f + 1) = krf + (1 - k)r(f + 1) = rf + r - kr$.

by $\overline{\mathcal{Q}} = \mathcal{F}_+^z = \{\overline{q}\}$ the space of all feasible capacity configurations, where $\overline{q} = (\overline{q}_1, ..., \overline{q}_z)$ is a vector of capacities, one for each potential entrant, which might result from stage-1 capacity decisions. Furthermore, let $\overline{q}_{-i} \in \mathcal{F}_+^{z-1}$ denote the capacity configuration of i's rivals, $\overline{Q} = \overline{q}$ total capacity, $\mathcal{A} = \{i \mid \overline{q}_i > 0\}$ and $n = \#\mathcal{A}$ the set and number of active firms at $\overline{q}$, respectively, and g any firm with the largest capacity. At stage 2 every $i \in \mathcal{A}$ knows $\overline{q}$.

We would like to compare the outcome of strategic market interaction through prices or quantities with the long-run competitive equilibrium, namely, the equilibrium of the industry when price-taking potential entrants make simultaneous capacity and quantity decisions. Unfortunately, the competitive equilibrium may not exist. (For nonexistence under U-shaped average cost, see Novshek, 1980, pp. 473-4, and Mas-Colell, Whinston, and Green, 1995, pp. 337-8). In fact, total supply $S(p)$ is indefinitely large at $p > r$, and zero at $p < r$. If at zero profits the firms are indifferent between entering or not, then $S(r) \in \mathcal{F}_+$: at $p = r$ the firms choose any feasible capacity and supply it entirely. Thus it can only be $S(r) = D(r)$ if $D(r) \in \mathcal{F}_+$.[4] In the text we conveniently avoid the nonexistence problem by restricting ourselves to demand curves such that $\frac{a-r}{b} \in \mathcal{F}_+$; thus the "competitive" price (p^*) and output (Q^*) are, respectively, $p^* = r$ and $Q^* = D(r)$. Furthermore, with $D(r) \in \mathcal{F}_+$ equilibria of our capacity and market games are much easier to characterize. (Equilibrium characterization for the $D(r) \notin \mathcal{F}_+$ case is carried out in Appendix B.)

We denote by $\overline{\mathcal{Q}}^* = \{\overline{q}^*\}$ the set of all capacity configurations with $D(r)$ active firms, each with unitary capacity, and by $\overline{\mathcal{Q}}^{**} = \{\overline{q}^{**}\}$ the set of all capacity configurations with $D(r) + 1$ active firms, each with unitary capacity. More formally, we have the following definition.

Definition 1. *Each $\overline{q}^*$ is such that $n^* = \overline{Q}^* = D(r)$, and each $\overline{q}^{**}$ is such that $n^{**} = \overline{Q}^{**} = D(r) + 1$.*

Hence $\overline{\mathcal{Q}}^*$ is the set of the least concentrated capacity configurations consistent with the long-run competitive capacity. By assumption, there are sufficiently many potential entrants for $\overline{q}^{**}$ to be feasible ($z > n^{**}$).

Further notation is now introduced. At any $\overline{q}$, let $p^w(\overline{q})$ and $Q^w(\overline{q})$ be, respectively, the market-clearing price and total output with price-taking firms: $p^w(\overline{q}) = P(\overline{Q})$ and $Q^w(\overline{q}) = \overline{Q}$ if $\overline{Q} \leq D(0)$, while $p^w(\overline{q}) = 0$ and $Q^w(\overline{q}) = D(0)$ if $\overline{Q} \geq D(0)$. Henceforth $\pi_i^w(\overline{q}) = (p^w(\overline{q}) - r)\overline{q}_i$ denotes i's market-clearing profit at $\overline{q}$ and $\pi_i^w(\overline{q}_i, \overline{q}_{-i}) = (p^w(\overline{q}_i, \overline{q}_{-i}) - r)\overline{q}_i$ denotes i's market-clearing profit as a function of $\overline{q}_i$, given $\overline{q}_{-i}$. If $\overline{q}_i$ were continuous, then concavity of $\pi_i^w(\overline{q}_i, \overline{q}_{-i})$ would follow straightforwardly from $D''(p) \leq 0$;

[4] Even so, some coordination is needed for the firms to exactly supply $D(r)$.

in our context, $\partial^2 \pi_i^w(\overline{q}_i, \overline{q}_{-i})/\partial \overline{q}_i^2 = -2b$ when $\overline{q}_i + \sum_{j \neq i} \overline{q}_j < D(0)$.

3 The capacity and price game

3.1 The price subgame

We characterize the equilibrium of the price subgame at any $\overline{q}$. Let p_{-i} be the prices of i's rivals and $d_i(p_i, p_{-i}, \overline{q})$ the demand facing i at prices (p_i, p_{-i}). The firms produce on demand, hence firm i's output is $q_i(p_i, p_{-i}, \overline{q}) = \min\{d_i(p_i, p_{-i}, \overline{q}), \overline{q}_i\}$. Under efficient rationing, $d_i(p_i, p_{-i}, \overline{q}) = \max\{0, D(p_i) - \sum_{j \neq i} \overline{q}_j\}$ when $p_i > p_j$ for any $j \neq i \in \mathbb{A}$. Let $\widetilde{q}_i = \widetilde{q}(\sum_{j \neq i} \overline{q}_j)$ be such that $\left[\partial[P(q_i + \sum_{j \neq i} \overline{q}_j)q_i]/\partial q_i\right]_{q_i = \widetilde{q}_i} = 0$ and let $\widetilde{\Pi}_i = P(\widetilde{q}_i + \sum_{j \neq i} \overline{q}_j)\widetilde{q}_i$. Note that, so long as $\widetilde{q}_i \leq \overline{q}_i$, $\widetilde{q}_i$ and $\widetilde{\Pi}_i$ are, respectively, firm i's (short-run) Cournot best response and revenue in the face of an output of $\sum_{j \neq i} \overline{q}_j$ by its rivals. With $D''(p) \leq 0$, $\widetilde{q}'(\cdot) < 0$ for $\sum_{j \neq i} \overline{q}_j < D(0)$. We also let $\widetilde{p}_i = \widetilde{p}(\sum_{j \neq i} \overline{q}_j)$ be such that $\left[d[p_i(D(p_i) - \sum_{j \neq i} \overline{q}_j)]/dp_i\right]_{p_i = \widetilde{p}_i} = 0$. Note that $\widetilde{p}_i$ is firm i's best price response to strategy profile p_{-i} so long as $\widetilde{p}_i \geq P(\overline{Q})$ and $\widetilde{p}_i \geq p_j$ for any $j \neq i$. It is $0 < \widetilde{p}_i \leq P(\sum_{j \neq i} \overline{q}_j)$; furthermore, $\max_i p_i = \widetilde{p}_g$ because $\widetilde{p}'(\cdot) < 0$ for $\sum_{j \neq i} \overline{q}_j < D(0)$. Clearly, $\widetilde{p}_i = P(\widetilde{q}_i + \sum_{j \neq i} \overline{q}_j)$ and hence $\widetilde{\Pi}_i = \widetilde{p}_i \widetilde{q}_i$. In our setting,

$$\widetilde{q}_i = \frac{a - b\sum_{j \neq i} \overline{q}_j}{2b}, \quad (1)$$

$$\widetilde{p}_i = \frac{a - b\sum_{j \neq i} \overline{q}_j}{2}. \quad (2)$$

Let $\pi_i(\overline{q})$ and $\Pi_i(\overline{q})$ be, respectively, i's expected profit and revenue at an equilibrium of the price subgame. The following result is easily established.

Lemma 1. *For any $i \in \mathcal{A}$, $\pi_i(\overline{q}) \geq \pi_i^w(\overline{q})$.*

Proof. The claim is obvious when $p^w(\overline{q}) = 0$. With $p^w(\overline{q}) = P(\overline{Q})$, by charging $p^w(\overline{q})$ firm i guarantees itself full capacity utilization, hence a profit of $\pi_i^w(\overline{q})$, regardless of p_{-i}. ■

We now see that, with $\overline{Q} \neq D(0)$, the market-clearing price obtains at an equilibrium of the price subgame provided individual capacities are small compared to industry capacity (for the symmetric case, see Vives, 1986).

Lemma 2. *A small $\overline{q}_g/\overline{Q}$ is necessary and sufficient for: (i) all prices equal to zero to be an equilibrium of the price subgame when $\overline{Q} > D(0)$;*

(ii) all prices equal to $P(\overline{Q})$ to be the equilibrium of the price subgame when $\overline{Q} < D(0)$.

Proof. (i) All prices equal to zero is an equilibrium if and only if $\sum_{j \neq g} \overline{q}_j \geq D(0)$ or, equivalently, $\overline{q}_g/\overline{Q} \leq 1 - D(0)/\overline{Q}$: for any $D(0)$ and $\overline{Q} \geq 2$, this condition will hold if $\overline{q}_g/\overline{Q}$ is sufficiently small.[5]

(ii) From concavity of $p_i(D(p_i) - \sum_{j \neq i} \overline{q}_j)$, each firm charging $P(\overline{Q})$ is an equilibrium if and only if

$$\left[\frac{\partial(p_i(D(p_i) - \sum_{j \neq i} \overline{q}_j)}{\partial p_i}\right]_{p_i = P(\overline{Q})} = \overline{q}_i + P(\overline{Q}) \left[D'(p)\right]_{p=P(\overline{Q})} \leq 0 \text{ for all } i \in \mathcal{A}, \tag{3}$$

that is, if and only if $\overline{q}_g \leq -P(\overline{Q}) \left[D'(p)\right]_{p=P(\overline{Q})}$. This can in turn be written as

$$\overline{q}_g/\overline{Q} \leq \eta_{p=P(\overline{Q})}, \tag{4}$$

where $\eta_{p=P(\overline{Q})}$ denotes absolute elasticity of $D(p)$ at a price of $P(\overline{Q})$. Uniqueness of equilibrium can be established similarly as in KS. ■

Note, incidentally, that, because of concavity of $p_i(D(p_i) - \sum_{j \neq i} \overline{q}_j)$ and since $\widetilde{p}\,'(\cdot) < 0$, ineq. (3) amounts to

$$\widetilde{p}_g \leq P(\overline{Q}). \tag{5}$$

where $\widetilde{p}_g = \widetilde{p}(\sum_{j \neq g} \overline{q}_j)$. A pure-strategy equilibrium (p.s.e.) does not exist when $\overline{Q} \geq D(0)$ and $\sum_{j \neq g} \overline{q}_j < D(0)$ or when $\overline{Q} < D(0)$ and $\widetilde{p}_g > P(\overline{Q})$. Then a mixed-strategy equilibrium (m.s.e.) exists; in fact, as we now see, all the sufficient conditions of Theorem 5 of Dasgupta and Maskin (1986) for equilibrium existence are satisfied. Let $\Pi_i(p_i, p_{-i}; \overline{q})$ denote firm i's expected revenue in terms of (p_i, p_{-i}), given $\overline{q}$. First, for any $i \in \mathcal{A}$, $\Pi_i(p_i, p_{-i}; \overline{q})$ is bounded and continuous in p_i, except at (p_i, p_{-i}) such that $p_i = p_j$ for some $j \neq i$ and $0 < D(p_i) - \sum_{j:p_j<p_i} \overline{q}_j < \sum_{j \neq i:p_j=p_i} \overline{q}_j + \overline{q}_i$, where $\Pi_i(\cdot)$ is weakly lower semi-continuous (at any such (p_i, p_{-i}), a slight reduction in p_i results in an upward jump in $\Pi_i(\cdot)$). Second, $\sum_{i \in \mathcal{A}} \Pi_i(\cdot)$ is continuous, hence upper semicontinuous, in (p_i, p_{-i}). As to the properties of m.s.e., since KS it has been known that, under duopoly, expected equilibrium revenue for the largest firm equals the revenue of the Stackelberg follower when

[5] Remaining equilibria have sufficiently many firms charging 0 so that $\sum_{j \neq i:p_j=0} \overline{q}_j \geq D(0)$ for any $i : p_i = 0$.

the rival supplies its entire capacity. While this property was subsequently extended to symmetric oligopoly (Vives, 1986), the following lemma (based on a claim by Boccard and Wauthy, 2000, subsequently further developed by De Francesco, 2003) establishes its generality.

Lemma 3. *At any $\overline{q}$ for which no p.s.e. exists, firm g's expected equilibrium revenue is* $\Pi_g(\overline{q}) = \widetilde{\Pi}_g = \widetilde{p}_g \widetilde{q}_g$, *where* $\widetilde{q}_g < \overline{q}_g$.

Proof. See De Francesco (2003). ■

Remark 1. Let $\pi_i^w(\overline{q}_i = \widetilde{q}_i, \overline{q}_{-i}) \equiv \pi_i^w(\widetilde{q}_i, \overline{q}_{-i})$, where $\pi_i^w(\widetilde{q}_i, \overline{q}_{-i}) = (P(\widetilde{q}_i + \sum_{j \neq i} \overline{q}_j) - r)\widetilde{q}_i$. Firm g's expected profit at a m.s.e., $\pi_g(\overline{q}) = \widetilde{p}_g \widetilde{q}_g - r\overline{q}_g$, can then be written $\pi_g(\overline{q}) = \pi_g^w(\widetilde{q}_g, \overline{q}_{-g}) - r(\overline{q}_g - \widetilde{q}_g)$.

3.1.1 Solving the entire game

Looking for subgame perfect equilibria of the capacity and price game, we begin ruling out any $\overline{q} \notin \overline{\mathcal{Q}}^*$.

Lemma 4. *Capacity configurations outside $\overline{\mathcal{Q}}^*$ cannot occur at an equilibrium of the capacity and price game.*

Proof. See Appendix A. ■

Now we see that any capacity configuration $\overline{q}^*$ is part of an equilibrium, with pricing on the equilibrium path depending on the size of $D(r)$.

Proposition 1 *At an equilibrium of the capacity and price game: (i) if $b \leq r$ (that is, $D(r) \geq (a - r)/r$), then the capacity configuration is any $\overline{q}^*$ and the firms are charging the competitive price r on the equilibrium path; (ii) if $b > r$ (that is, $D(r) < (a - r)/r$), then the capacity configuration is any $\overline{q}^*$ and the firms randomize over prices on the equilibrium path.*

Proof. (i) Coherently with our notation, we let $\widetilde{p}_i^* = \widetilde{p}(\sum_{j \neq i} \overline{q}_j^*)$ and $\widetilde{p}_i^{**} = \widetilde{p}(\sum_{j \neq i} \overline{q}_j^{**})$. By substitution into eq. (2), $\widetilde{p}_i^* = (r + b)/2$, hence $\widetilde{p}_i^* \leq r$ if and only if $b \leq r$. This being so, a p.s.e. for the price subgame obtaines at $\overline{q}^*$ and $\pi_i(\overline{q}^*) = 0$. Any $i \in \mathcal{A}^*$ [6] has made a best capacity response to $\overline{q}_{-i}^*$, no matter whether $\widetilde{p}_i^* \gtreqless P(\overline{Q}^* + 1)$. If $\widetilde{p}_i^* > P(\overline{Q}^* + 1)$, then a m.s.e. obtains when i deviates to $\overline{q}_i \geq 2$, resulting in $\pi_i(\overline{q}_i, \overline{q}_{-i}^*) = \widetilde{p}_i^* \widetilde{q}_i^* - r\overline{q}_i$. This is negative because $\widetilde{p}_i^* \leq r$ and $1 \leq \widetilde{q}_i^* < 2 \leq \overline{q}_i$. If $\widetilde{p}_i^* \leq P(\overline{Q}^* + 1)$, then deviating to $\overline{q}_i = 2$ leads to a p.s.e., hence to a loss. A fortiori losses would result from deviating to any $\overline{q}_i > 2$ entailing a p.s.e.. If choosing $\overline{q}_i > 2$

[6] Consistent with our terminology, $\mathcal{A}^* = \{i \mid \overline{q}_i^* = 1\}$.

leading to a m.s.e., then $\pi_i(\overline{q}_i, \overline{q}^*_{-i}) = \widetilde{p}^*_i \widetilde{q}^*_i - r\overline{q}_i < 0$ since $\widetilde{p}^*_i \leq P(\overline{Q}^* + 1) < r$ and $\widetilde{q}^*_i < \overline{q}_i$. Finally, at $\overline{q}^*$ any firm $u \notin \mathcal{A}^*$ has made a best response. Suppose u deviate to $\overline{q}_u = 1$. The resulting configuration $(\overline{q}_u = 1, \overline{q}^*_{-u})$ can be denoted by $\overline{q}^{**}$: thus $\pi_u(\overline{q}_u = 1, \overline{q}^*_{-u}) = \pi_i(\overline{q}^{**})$, $\pi_i(\overline{q}^{**})$ being the equilibrium payoff of any $i \in \mathcal{A}^{**}$ at $\overline{q}^{**}$. Obviously $\pi_i(\overline{q}^{**}) < 0$ if a p.s.e. obtains at $\overline{q}^{**}$. If a m.s.e. obtains, then $\pi_i(\overline{q}^{**}) = \widetilde{p}^{**}_i \widetilde{q}^{**}_i - r$; this is negative because $\widetilde{p}^{**}_i = r/2$ and $\widetilde{q}^{**}_i < 1$. Things would be even worse for u if entering with $\overline{q}_u > 1$.

(ii) A m.s.e. obtains at $\overline{q}^*$, hence $\pi_i(\overline{q}^*) = \widetilde{p}^*_i \widetilde{q}^*_i - r > 0$. Every $i \in \mathcal{A}^*$ has replied optimally because deviating to $\overline{q}_i > 1$ raises cost without affecting subgame equilibrium (hence expected revenue). Any $u \notin \mathcal{A}^*$ has also made a best response: if deviating to $\overline{q}_u = 1$, $\pi_u(\overline{q}_u, \overline{q}^*_{-u}) = \pi_i(\overline{q}^{**}) < 0$ at the m.s.e. of the resulting subgame. ∎

The symmetric m.s.e. obtaining at $\overline{q}^*$ when $b > r$ is easily computable (see Vives, 1999, pp. 130-1). There is an atomless equilibrium distribution, $\phi(p)$, over a compact support $S = [\underline{p}^*, \overline{p}^*]$; since expected revenue is $\widetilde{\Pi}^*_i = \widetilde{p}^*_i \widetilde{q}^*_i$, we have $\overline{p}^* = \widetilde{p}^*_i$ and $\underline{p}^* = \widetilde{\Pi}^*_i$. Finally, $\widetilde{\Pi}^*_i = p[\phi^{n-1}(\frac{a-p}{b} - (n^* - 1)) + (1 - \phi^{n-1})]$ for any $p \in S$, hence $\phi(p) = \sqrt[n^*-1]{\frac{b(\widetilde{\Pi}^*_i - p)}{p(a-p-bn^*)}}$.

To summarize: at any solution of the game, total capacity equals the long-run competitive output $D(r)$, each active firm has the minimum feasible size, and the competitive price emerges on the equilibrium path so long as $D(r) \geq (a-r)/r$, or else the firms randomize over prices. Note, incidentally, that what really matters is the size of $D(r)$ *relative* to the firm minimum efficient size ($\overline{\alpha}$), which we normalized to 1.

4 The capacity and quantity game

We now determine Cournot equilibrium with entry. Cournot competitors are simultaneous quantity setters that take the price equal to the demand price of the total output brought to the market. In a long-run setting, Cournot competitors are simultaneous capacity setters. Under KS's assumption of perfect capacity divisibility, capacity is obviously set equal to the planned output. However, with indivisibility, equilibrium output might be less than capacity.

We keep the two-stage setup: potential entrants choose capacity and active firms subsequently choose output. A static setup - where the firms choose simultaneously a capacity and quantity pair - is perhaps more akin to the normal way of portraying long-run Cournot competition. Yet, the two-stage setup is not fundamentally misleading: as will be shown in Appendix C, any (subgame-perfect) equilibrium outcome of the two-stage game constitutes an equilibrium of the static capacity and quantity game.

As can easily be checked, when the equilibrium of the quantity subgame at $\overline{q}$ involves an internal maximum for each of the n active firms, then the equilibrium is symmetric, with $q_i = \frac{a}{b(n+1)}$ and $\Pi_i = \frac{a^2}{b(n+1)^2}$. Taking stock of this, we now look for equilibria of the capacity and quantity game. We preliminarily dispose of any $\overline{q}$ such that $\overline{q}_g > 1$.

Lemma 5. *No $\overline{q}$ where $\overline{q}_g > 1$ can occur at an equilibrium of the capacity and quantity game.*

Proof. See Appendix A. ■

We can now provide a full characterization of equilibria for the capacity and quantity game.

Proposition 2 *At an equilibrium of the capacity and quantity game: (i) if $b \leq r$, then the capacity configuration is any $\overline{q}^*$ and the firms produce their capacity on the equilibrium path, which results in the competitive price r; (ii.a) if $b > r > \frac{a^2 b}{(a+2b-r)^2}$, then the capacity configuration is any $\overline{q}^*$ and the firms produce below capacity on the equilibrium path; (ii.b) if $b > \frac{a^2 b}{(a+2b-r)^2} \geq r$, then the capacity configuration is any $\overline{q}^\S$ such that $n^\S = \overline{Q}^\S$ and $n^\S \leq -1 + \frac{a\sqrt{br}}{br} < n^\S + 1$, and the firms produce below capacity on the equilibrium path.*

Proof. (i) Since $b \leq r$, $\partial[q_i P(q_i + \sum_{j \neq i} \overline{q}_j^*)]/\partial q_i \geq 0$ at $q_i = 1$: at $\overline{q}^*$, the firms produce their capacity at the equilibrium of the quantity subgame. Any $i \in \mathcal{A}^*$ has made a best capacity response. To this effect, note that $\widetilde{q}_i^* = \frac{b+r}{2b}$, where $\widetilde{q}_i^*$ is i's unconstrained (short-run) best quantity response to an output of $\sum_{j \neq i} \overline{q}_j^*$. Let $k \in \mathcal{F} : k - 1 < \widetilde{q}_i^* \leq k$. As one can check, if i deviates to $\overline{q}_i = k$, then, at the subgame equilibrium, $q_i = \widetilde{q}_i^*$ and $q_j = \overline{q}_j^* = 1$ for any $j \neq i \in \mathcal{A}^*$; thus $Q > D(r)$ and $p = P(Q) < r$. With $k \geq 3$, deviating to any $\overline{q}_i \in \{2, ..., k-1\}$ results in $q_i = \overline{q}_i$ and $q_j = \overline{q}_j^* = 1$ for any $j \neq i \in \mathcal{A}^*$: again $Q > D(r)$. A loss would also be faced by any $u \notin \mathcal{A}^*$ if entering. Deviating to $\overline{q}_u = 1$ results in a configuration $\overline{q}^{**}$. If $b \leq r/2$, then a boundary solution obtains for the quantity subgame, so that $Q = \overline{Q}^* + 1 > D(r)$. As for a deviation to $\overline{q}_u > 1$ one can adapt a previous argument to see that $Q > D(r)$. If $r/2 < b \leq r$, an internal solution obtains if deviating to $\overline{q}_u \geq 1$: thus total output is $\frac{n^{**}}{n^{**}+1}\frac{a}{b} = \frac{a(a+b-r)}{b(a+2b-r)} > D(r)$.

As to configurations outside $\overline{\mathcal{Q}}^*$, in view of Lemma 5 we just need to focus on any $\overline{q} : n = \overline{Q} \neq D(r)$. Any $\overline{q} : n = \overline{Q} < D(r)$ is immediately dismissed since any inactive firm would profit from entering with $\overline{q}_u = 1$.[7] As to any $\overline{q} : n = \overline{Q} = D(r) + l$ (with $l \geq 1$), note that an internal solution obtains for the quantity subgame if $b(1 + l) > r$. (The case of a boundary solution is trivial.) Equilibrium revenue is then $\frac{a^2}{b(n+1)^2} = \frac{a^2 b}{[a+b(1+l)-r]^2} < b \leq r$: any active firm makes losses.

(ii.a) With $b > r$, an internal solution for the quantity subgame obtains at $\overline{q}^*$ and $\pi_i(\overline{q}^*) > 0$. A best response has been made at stage 1 by any $i \in \mathcal{A}^*$: deviating to $\overline{q}_i > 1$ would raise cost without affecting subgame equilibrium. As to any $u \notin \mathcal{A}^*$, entering would afford revenue $\Pi_i(\overline{q}^{**}) = a^2 b/(a + 2b - r)^2 < r$, no matter $\overline{q}_u$. Other configurations where active firms have one unit of capacity are easily dismissed: we have just seen that $\pi_i(\overline{q}^{**}) < 0$; then, a fortiori $\pi_i(\overline{q}) < 0$ for any $\overline{q} : n = \overline{Q} > D(r) + 1$.

(ii.b) Note that $n^{\S}$ is the largest integer solution of $\frac{a^2}{b(n+1)^2} \geq r$. Thus, for any $u \notin \mathcal{A}^{\S}$, staying out is the best response at $\overline{q}^{\S}$. Any $i \in \mathcal{A}^{\S}$ has also made a best response because a capacity increase would just raise costs. Other $\overline{q}$s where active firms have one unit of capacity are easily dismissed. ■

By comparing Propositions 1 and 2 we then see that the outcomes of Cournot and price competition do coincide so long as $D(r)$ is sufficiently large, the competitive price then emerging in either setting: on the equilibrium path, there will be $D(r)$ active firms, each producing its unitary capacity. With $D(r)$ not that large, the two settings yield quite different outcomes. On the equilibrium path of the capacity and price game there are still $D(r)$ active firms, each with unitary capacity, but they are now playing a m.s.e. of the price subgame. Under Cournot competition, active firms are producing less than their unitary capacity on the equilibrium path; quite importantly, there may be much more than $D(r)$ active firms (as illustrated by the last example below), hence the level of total capacity may exceed competitive capacity.

Examples. 1. $a = 15$, $b = 1$, and $r = 2$. At an equilibrium of either game, $n = \overline{Q} = D(r) = 13$, and the competitive price r obtains.

2. $a = 10.5$, $b = 3$, and $r = 1.5$. At an equilibrium of either game, $n = \overline{Q} = D(r) = 3$, but stage-2 equilibrium variables differ. In the price subgame, $\Pi_i = \widetilde{\Pi}_i^* = 1.6875$, and $\phi(p) = \sqrt[2]{\frac{3(1.6875-p)}{p(1.5-p)}}$ over $S = [1.6875, 2.25]$; in the quantity subgame, $q_i = .875$, $p = 2.625$, and $\Pi_i = 2.296875$.

[7] With $\overline{Q} = \overline{Q}^* - 1$, entry would result in zero profits. Any such $\overline{q}$ is disposed of if, at zero profit, entering is strictly preferred to not entering.

3. $a = 17$, $b = 2$, and $r = 1$. In the capacity and price game, $n = \overline{Q} = D(r) = 8$, $\Pi_i = \widetilde{\Pi}_i^* = 1.125$, and $\phi(p) = \sqrt[7]{\frac{2(1.125-p)}{p(1-p)}}$ over $S = [1.125, 1.5]$. Capacity is much larger at a Cournot equilibrium: $n = \overline{Q} = \overline{n}^{\S} = 11$, $q_i = 17/24$, $p = 17/12$, and $\Pi_i = 289/288$. $\diamond$

5 The role of capacity indivisibility

With nonconvexities in long-run costs, "excess capacity" - namely, active firms producing less than the average cost minimizing output - is a common feature of models of imperfect competition with entry (in the Cournot setting, see Novshek, 1980). On the other hand, the possibility of Cournot and Bertrand-Edgeworth competition *exactly* yielding the (long-run) competitive price and total output constitutes one distinctive feature of our model. This result relies on the discontinuities in the cost function. Suppose, as before, a unique technology to be available, but now let capacity be a continuous choice variable: then the long-run cost function would be $c(q_i) = rq_i$ for any $q_i \in \mathbb{R}_+$. Consider any $\overline{q}$ such that $\overline{Q} = D(r)$ and $p = P(\overline{Q}) = r$ at an equilibrium of both the price and quantity subgame. Clearly, any active firm has not made a best capacity response: by reducing capacity the new market-clearing price would be raised above r, thus allowing for positive profits at the price or quantity subgame. In contrast, there is no such opportunity in our model because, at the candidate equilibrium, the firms are endowed with the minimum capacity that is technically feasible.

Although our model has been developed assuming availability of a single technology, one might allow for a plurality of technologies and still end up with the long-run competitive outcome at an equilibrium of either game. To see this in the easiest way, let another technology, β, be also available, besides α. Similar to α, if choosing β the capacity choice set will be $\mathcal{C}^{(\beta)} = \overline{\beta}\mathcal{F}_+$ and average cost will be constant at r' under full capacity utilization. We take β to be a "less mechanized" technology, allowing for a lower minimum capacity ($\overline{\beta} < \overline{\alpha} = 1$). Furthermore, we let $r'\overline{\beta} < r < r'$: the average-cost minimizing technology is α, but β is cheaper at a sufficiently low output. It might easily be verified that, at the long-run competitive equilibrium, technology α is the one actually chosen, while price and capacity are $p^* = r$ and $\overline{Q}^* = D(r)$, respectively. Now, so long as $b \leq r$, this same outcome *may* still arise at an equilibrium of either game, with each active firm endowed with one unit of capacity. On reflection, this is actually the case when, at

$\overline{q}^*$,[8] it does *not* pay any $i \in \mathcal{A}^*$ to deviate to technology β and install any capacity $\overline{q}_i \in \overline{\beta}[1,...,k] < 1$, where $k < 1/\overline{\beta} < k+1$.[9] Note that, with price competition as well as Cournot competition, at the resulting subgame the deviant will sell $\overline{q}_i$ at the new market-clearing price, $r + b(1 - \overline{q}_i)$. Then such a move will lead to losses if $r + b(1 - \overline{q}_i) < r'$. It follows that deviating to any feasible $\overline{q}_i < 1$ is unprofitable so long as $\overline{\beta} > [b - (r' - r)]/b$.

Finally, it must be emphasized that capacity indivisibility is crucial for the possibility of Bertrand-Edgeworth competition not yielding the Cournot outcome, in contrast to KS. This can be seen by showing that, with (weak) convexity in costs, the result established by KS for a duopoly will always hold. Let $D''(p) \leq 0$ and $c''(\overline{q}_i) \geq 0$. Most importantly, no $\overline{q}$ involving a m.s.e. of the price subgame can arise at an equilibrium of the capacity and price game. Indeed, at any such $\overline{q}$, $\pi_g(\overline{q}) = \pi_g^w(\widetilde{q}_g, \overline{q}_{-g}) - [c(\overline{q}_g) - c(\widetilde{q}_g)]$, where $\widetilde{q}_g < \overline{q}_g$. Then g has not made a best capacity response: by Lemma 1, with capacity $\widetilde{q}_g$ it would earn at least $\pi_g^w(\widetilde{q}_g, \overline{q}_{-g})$. Furthermore, unlike under capacity indivisibility, profits are positive at an equilibrium of the entire game, no matter the number of firms: if not, any active firm would profit from deviating to a lower capacity and then charging the new market-clearing price. Thus the equilibrium price is higher than unit cost. It follows that all firms are active at an equilibrium of the entire game: if not, any inactive firm would profit from entering with a sufficiently small capacity and then charging the market-clearing price.

The equilibrium of the (Cournot) capacity and quantity game is a profile of capacity-quantity decisions - call it $\overline{q}^c$ - such that

$$\frac{d}{d\overline{q}_i}\left(P(\overline{q}_i + \sum_{j \neq i} \overline{q}_j)\overline{q}_i\right) = c'(\overline{q}_i) \quad \text{for all } i \in \mathcal{Z}. \tag{6}$$

Any Cournot equilibrium is symmetric; furthermore, provided $P(0) > c'(0)$, equilibrium existence is guaranteed by (weak) concavity of demand and (weak) cost convexity. Of course, the left-hand side of (6) is positive at the Cournot equilibrium, hence $\widetilde{q}_i^c > \overline{q}_i^c$, where $\widetilde{q}_i^c$ is firm i's (short-run) unconstrained best response in the face of a total output of $\sum_{j \neq i} \overline{q}_j = (z-1)\overline{q}_i^c$: on the equilibrium path, $q_i^c = \overline{q_i}^c$ for any $i \in \mathcal{Z}$ and $p = P(\overline{Q}^c)$.

We can now establish the Cournot outcome of the capacity and price game (see also Boccard and Wauthy, 2000 and 2004).

[8] As before, $\overline{q}^*$ is such that $\overline{q}_i^* = \overline{\alpha} = 1$ for each $i \in \mathcal{A}^*$ and $n^* = D(r)$.

[9] Installing any $\overline{q}_i \geq 1$ while deviating to β is immediately discarded.

Proposition 3 *At an equilibrium of the capacity and price game, the capacity configuration is $\overline{q}^c$ and all firms charge $P(\overline{Q}^c)$ on the equilibrium path.*

Proof. Among configurations involving a p.s.e. for the price subgame, consider any $\overline{q}$ such that $d[P(\overline{q}_i + \sum_{j \neq i} \overline{q}_j)\overline{q}_i]/d\overline{q}_i > c'(\overline{q}_i)$ for some firm i. Any such i has not made a best capacity response: by marginally increasing $\overline{q}_i$ and then charging the market-clearing price, it would raise revenues more than cost. A similar argument disposes of any $\overline{q}$ where $d[P(\overline{q}_i + \sum_{j \neq i} \overline{q}_j)\overline{q}_i]/d\overline{q}_i < c'(\overline{q}_i)$ for some i. To check that $\overline{q}^c$ is instead an equilibrium, let i deviate to $\overline{q}_i > \overline{q}_i^c$. If $\overline{q}_i \leq \widetilde{q}_i^c$, then, by concavity of $P(\overline{q}_i + \sum_{j \neq i} \overline{q}_j)\overline{q}_i - c(\overline{q}_i)$, i's profit would be less than at $\overline{q}^c$ at the p.s.e. of the price subgame. Prospects are even worse if $\overline{q}_i > \widetilde{q}_i^c$: a m.s.e. would then obtain for the price subgame, with i's expected revenue fixed at $P(\widetilde{q}_i^c + \sum_{j \neq i} \overline{q}_j^c)\widetilde{q}_i^c$, no matter $\overline{q}_i$. Next consider a deviation to $\overline{q}_i < \overline{q}_i^c$. As $\overline{q}_i$ decreases, $\widetilde{p}_j$ increases at rate $-1/[2D'(\cdot) + P(\cdot)D''(\cdot)]$ for any $j \neq i$, whereas the market-clearing price $P(\overline{q}_i + \sum_{j \neq i} \overline{q}_j^c)$ increases at the faster rate $-P'(\cdot)$. Therefore, $\widetilde{p}_j < P(\overline{q}_i + \sum_{j \neq i} \overline{q}_j^c)$ for any $j \neq i$: the price subgame has still a p.s.e. and we can use the previous argument to conclude that i's profits decrease. ■

6 Conclusion

The paper has analyzed entry and strategic market interaction through prices or quantities as a two-stage game where many potential entrants are facing a discrete capacity choice set at stage 1. Whether the firms are price setters or Cournot quantity setters, the equilibrium outcome has been seen to depend on the market size at the long-run competitive equilibrium: with a sufficiently large market, the competitive price emerges in either game; failing the large market condition, the competitive outcome does not arise and equilibrium outcomes are quite different in the two strategic settings.

Our two main results - the possibility of either game yielding exactly the competitive outcome and the possibility of the capacity and price game not yielding the Cournot outcome - rely on the discontinuities in the cost function, which in turn have been derived from capital indivisibility under the simplifying assumption of availability of a single technology. Checking how these results should be qualified assuming a plurality of available technologies - which would mitigate to some extent the degree of capacity indivisibility - is a task that we leave to future research.

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APPENDIX
A. Remaining proofs

Proof of Lemma 4. Let $\overline{q}_i^\dagger = argmax_{\overline{q}_i} \pi_i^w(\overline{q}_i, \overline{q}_{-i})$ when $\overline{q}_i$ is viewed as a continuous variable: $\overline{q}_i^\dagger = 0.5[D(r) - \sum_{j \neq i} \overline{q}_j]$ when $\sum_{j \neq i} \overline{q}_j \leq D(r)$. One can also check that

$$\pi_i^w(\overline{q}_i, \overline{q}_{-i}) = \pi_i^w(\overline{q}_i^\dagger, \overline{q}_{-i}) - b\left(\overline{q}_i - \overline{q}_i^\dagger\right)^2. \tag{7}$$

Now, let $\overline{\mathcal{Q}} \setminus \overline{\mathcal{Q}}^*$ denote the complementary set of $\overline{\mathcal{Q}}^*$ in the space $\overline{\mathcal{Q}}$ of capacity configurations. It is usefully partitioned into five subsets.

(a) $\{\overline{q} \mid \overline{Q} < \overline{Q}^*\}$. At any such $\overline{q}$, any $u \notin \mathcal{A}$ has not replied optimally, for it would profit from deviating to $\overline{q}_u = 1$ and then charging $p_u = P(\overline{Q}+1)$.[10]

(b) $\{\overline{q} \mid \overline{Q} = \overline{Q}^*; \overline{q}_g > 1\}$. No matter whether the price subgame has a p.s.e. or a m.s.e. at $\overline{q}$, firm g would benefit from deviating to $\overline{q}_g - 1$. In the former case this is immediate: $\pi_i(\overline{q}) = 0$ for any $i \in \mathcal{A}$, whereas g would profit from deviating to $\overline{q}_g - 1$ and then charging $P(\overline{Q}^* - 1) > r$. In the latter case, $\widetilde{p}_g > r$ and $\pi_g(\overline{q}) = \widetilde{\Pi}_g - r\overline{q}_g$. Then there are two possibilities: either $\widetilde{p}_g \geq P(\overline{Q}^* - 1)$ or $\widetilde{p}_g < P(\overline{Q}^* - 1)$. If $\widetilde{p}_g \geq P(\overline{Q}^* - 1)$, then deviating to $\overline{q}_g - 1$ would raise g's expected profit at least to $\widetilde{\Pi}_g - r(\overline{q}_g - 1)$: since rivals can produce $\sum_{j \neq g} \overline{q}_j$ at most, firm g will sell at least $\widetilde{q}_g = D(\widetilde{p}_g) - \sum_{j \neq g} \overline{q}_j \leq \overline{q}_g - 1$ when charging $\widetilde{p}_g$. If $\widetilde{p}_g < P(\overline{Q}^* - 1)$, then $\overline{q}_g - 1 < \widetilde{q}_g < \overline{q}_g$. The capacity reduction is then conveniently decomposed into two virtual steps: a reduction from $\overline{q}_g$ to $\widetilde{q}_g$ and then from $\widetilde{q}_g$ to $\overline{q}_g - 1$. By Lemma 1, it suffices to prove that g's profit would rise if, at each step, g were to charge the (short-run) market-clearing price. Assuming so, then g's profit would rise to $\pi_g^w(\widetilde{q}_g, \overline{q}_{-g})$ at the first step. At the second step, g's profit would become $\pi_g^w(\overline{q}_g - 1, \overline{q}_{-g})$: by eq. (7), this is larger than $\pi_g^w(\widetilde{q}_g, \overline{q}_{-g})$ because $\overline{q}_g^\dagger \leq \overline{q}_g - 1 < \overline{q}_g$ at any $\overline{q} : n < \overline{Q} = \overline{Q}^*$.

(c) $\{\overline{q} \mid \overline{Q} \geq \overline{Q}^* + 1; \widetilde{p}_g \leq P(\overline{Q})\}$. Any such $\overline{q}$ has a p.s.e., hence $\pi_i(\overline{q}) < 0$ for any $i \in \mathcal{A}$ given that $P(\overline{Q}) < r$.

(d) $\{\overline{q} \mid \overline{Q} = \overline{Q}^* + 1; \widetilde{p}_g > P(\overline{Q})\}$. Any such $\overline{q}$ involves a m.s.e.. Any $\overline{q} : \overline{q}_g > 1$ is dismissed as in part (b) above. As to the remaining subset $\{\overline{q}^{**}\}$, note that $\widetilde{p}_i^{**} = r/2$, hence $\pi_i(\overline{q}^{**}) = \widetilde{p}_i^{**}\widetilde{q}_i^{**} - r < \widetilde{p}_i^{**} - r < 0$.

(e) $\{\overline{q} \mid \overline{Q} > \overline{Q}^* + 1; \widetilde{p}_g > P(\overline{Q})\}$. Any such $\overline{q}$ has a m.s.e.. In this region, at any $\overline{q} : n = \overline{Q}$, it is $\pi_i(\overline{q}) = \widetilde{p}_i\widetilde{q}_i - r < 0$: in fact, $\widetilde{q}_i < \overline{q}_i = 1$ and

[10] With $\overline{Q} = \overline{Q}^* - 1$, this would result in zero profits if the resulting subgame has a p.s.e.. Any such $\overline{q}$ is disposed of if, at zero profit, entering is strictly preferred to not entering.

$\widetilde{p}_i < r$ because $\widetilde{p}_i = P(\widetilde{q}_i + \sum_{j \neq i} \overline{q}_j)$ and $\sum_{j \neq i} \overline{q}_j \geq \overline{Q}^* + 1$. Turn now to any $\overline{q} : \overline{q}_g > 1$. If $\widetilde{p}_g > P(\overline{Q} - 1)$, then we can argue as in (b). If $\widetilde{p}_g \leq P(\overline{Q} - 1)$, then $\widetilde{p}_g < r$ given that $P(\overline{Q} - 1) < r$: along with $\widetilde{q}_g < \overline{q}_g$, this reveals that $\pi_g(\overline{q}) = \widetilde{p}_g \widetilde{q}_g - r\overline{q}_g < 0$. ■

Proof of Lemma 5. At configurations such that $\overline{Q} < D(r)$ it would pay any inactive firm to deviate to $\overline{q}_u = 1$. Remaining configurations are partitioned according to whether $\overline{Q} = D(r)$ or $\overline{Q} > D(r)$.

(a) *Configurations such that* $\overline{Q} = D(r)$.

Let $G = \#\{i : \overline{q}_i = \overline{q}_g\}$. Configurations such that $Q = D(r)$ at the equilibrium of the quantity subgame are immediately disposed of: any firm g would profit from deviating to $\overline{q}_g - 1$, which would raise the market price above r. So we turn to $\overline{q}$s such that $q_g < \overline{q}_g$.[11] Denote by l any of the next to the largest firm(s). One possibility is that, at the equilibrium of the quantity subgame, firms l too reach a capacity-unconstrained maximum in revenues: then $q_l = q_g \leq \overline{q}_l \leq \overline{q}_g - 1$. In such an event, firm g would clearly profit from deviating to $\overline{q}_l$, which would just reduce its cost. Alternatively, it may be that $q_j = \overline{q}_j$ for any $j : \overline{q}_j < \overline{q}_g$: then, as one can check, $q_g = \frac{r + bG\overline{q}_g}{b(1+G)}$. One possible reason why such $\overline{q}$ cannot occur at an equilibrium of the entire game is that it might pay any u to deviate to $\overline{q}_u = 1$. If u so deviates, then $q_g < \overline{q}_g$ at the equilibrium of the new quantity subgame. Note, also, that $n < n^*$, where n is the number of firms at $\overline{q}$, with $n = n^* - 1$ if and only if $\overline{q}_g = 2$ and $G = 1$. Therefore, at the new quantity subgame - where there are $n + 1$ firms - an internal symmetric equilibrium obtains if and only if $b > r$ and $n = n^* - 1$. This being the case, u's profit will be $\frac{a^2}{b(n^*+1)^2} - r > 0$. In every other circumstance, it will instead be $q_u = 1$. Now, if at least firms l - along with firms g - reach a capacity-unconstrained maximum in revenues at the new subgame equilibrium, then, at this equilibrium, $Q < D(r)$ given that $\overline{Q} = D(r)$ and $q_l = q_g \leq \overline{q}_g - 1$: firm u will earn the positive profit $P(Q) - r$. The remaining case is when, after u's deviation, $q_j = \overline{q}_j$ for any $j : \overline{q}_j < \overline{q}_g$. In this case, $q_g = \frac{r + bG\overline{q}_g - b}{b(1+G)}$, $P(Q) = \frac{r + bG\overline{q}_g - b}{1+G}$, and $\pi_u = \frac{r + bG\overline{q}_g - b}{1+G} - r$: hence $\pi_u \geq 0$ so long as $\overline{q}_g \geq \frac{r}{b} + \frac{1}{G}$. If instead $\overline{q}_g < \frac{r}{b} + \frac{1}{G}$, then it can be seen that it pays any g to deviate to $\overline{q}_g - 1$. Denote by $\overline{q}'$ the capacity configuration after g's deviation to $\overline{q}_g - 1$ and by Q', $P(Q')$, and $\pi_g(\overline{q}')$, respectively, the total quantity, market price, and g's profit at the resulting equilibrium of the quantity subgame. It must preliminarily be seen that all firms are now producing their capacity given that $\overline{q}_g < \frac{r}{b} + \frac{1}{G}$. This is obvious as for any j :

[11] As can easily be checked, $q_g < \overline{q}_g$ when $Q < \overline{Q}$ at an equilibrium of the quantity subgame.

$\overline{q}_j < \overline{q}_g$, so we must prove that $q_r = \overline{q}_r$ for any $r \neq g : q_r = \overline{q}_g$. Consistently with our notation, let $\widetilde{q}_r$ denote firm r's (capacity unconstrained) short-run best quantity response when all the other firms are believed to produce their capacity: $\widetilde{q}_r = \frac{r+b\overline{q}_g+b}{2b} \geq \overline{q}_g$ if and only if $\overline{q}_g \leq (r/b)+1$, which in its turn holds true given that $\overline{q}_g < \frac{r}{b}+\frac{1}{G}$. As $Q' = D(r)-1$, it will be $P(Q') = b+r$. Thus $\pi_g(\overline{q}') = b\overline{q}_g - b$, which is higher than g's initial profit $\frac{(r+bG\overline{q}_g)^2}{b(1+G)^2} - r\overline{q}_g$ if and only if

$$b(1+G)^2\overline{q}_g(b+r) - b^2(1+G)^2 - (r+bG\overline{q}_g)^2 > 0. \tag{8}$$

With $\overline{q}_g = 2$ and $G \in \{1,2\}$, validity of (8) is established by substitution and taking account of $\overline{q}_g > r/b$ and $\overline{q}_g < r/b + 1/G$.[12] For all remaining cases, recall that $b(1+G)\overline{q}_g > r + bG\overline{q}_g$, as $q_g < \overline{q}_g$. Therefore, we would be done by establishing the following, more restrictive inequality:

$$(r+bG\overline{q}_g)(1+G)(b+r) - b^2(1+G)^2 - (r+bG\overline{q}_g)^2 > 0. \tag{9}$$

Letting $h = \frac{r}{b}+\frac{1}{G}-\overline{q}_g$ and recalling that $q_g = \frac{r+bG\overline{q}_g}{b(1+G)}$, ineq. (9) will lead to

$$q_g > \frac{1+G}{G(1+h)}. \tag{10}$$

Given that the right-hand side is less than 2 and $q_g > \overline{q}_g - 1$, ineq. (10) is certainly met if $\overline{q}_g > 2$. As to the remaining case of $\overline{q}_g = 2$ and $G > 2$, $q_g = \frac{r+2bG}{b(1+G)} > \frac{1+G}{G}$, given that $\overline{q}_g = 2 < \frac{r}{b}+\frac{1}{G}$.

(b) *Configurations such that* $\overline{Q} > D(r)$.

We can restrict ourselves to $\overline{q}$s such that, at the equilibrium of the quantity subgame, $Q < D(r)$, $\overline{q}_g - 1 < q_g < \overline{q}_g$ and $q_j = \overline{q}_j$ for any $j : \overline{q}_j < \overline{q}_g$ (other configurations are dismissed by drawing on arguments made in part (a)). It must preliminarily be seen that it is necessarily $G > \Delta$, where $\Delta = \overline{Q} - D(r)$. In fact, at the $\overline{q}$s under concern, $Q = \sum_{j:q_j<\overline{q}_g} \overline{q}_j + Gq_g = D(r) + \Delta - G\overline{q}_g + Gq_g > D(r) + \Delta - G\overline{q}_g + G(\overline{q}_g - 1) = D(r) + \Delta - G$, hence it has to be $G > \Delta$ in order for $Q < D(r)$.

One possible reason why any such $\overline{q}$ is ruled out as an equilibrium is that any inactive firm might profit from entering with $\overline{q}_u = 1$. Let $n = J + G$ denote the number of firms at $\overline{q}$, where $J = \#\{j : \overline{q}_j < \overline{q}_g\}$. It should be understood that, at the configurations under concern, $n \leq n^* - 1$, with

[12] Inequality $\overline{q}_g > r/b$ follows from the fact that, at the equilibrium of the quantity subgame at $\overline{q}$, $q_g < \overline{q}_g$ and $q_j = \overline{q}_j$ for any $j : \overline{q}_j < \overline{q}_g$.

$n = n^* - 1$ when $\Delta = G - 1$ and $\overline{q}_g = 2$.[13] Thus the subgame originating from u's deviation has an internal equilibrium if and only if $b > r$ and $n = n^* - 1$, in which case u's profit will be $\frac{a^2}{b(n^*+1)^2} - r > 0$, just as when $\overline{Q} = D(r)$. If not, it will be $q_u = 1$. Then there are two possibilities, similarly to when $\overline{Q} = D(r)$. It may be that, besides firms g, firms l also reach a capacity-unconstrained maximum in revenues at the new subgame equilibrium. Then total output will be not higher than $D(r)$ and firm u will thus earn nonnegative profits.[14] Or it may be that, at the new subgame equilibrium, $q_j = \overline{q}_j$ for any $j : \overline{q}_j < \overline{q}_g$. Then $q_g = \frac{r - b\Delta + bG\overline{q}_g - b}{b(1+G)}$, the price is $\frac{r-b\Delta+bG\overline{q}_g-b}{1+G}$ and firm u earns $\frac{r-b\Delta+bG\overline{q}_g-b}{1+G} - r$. Thus it pays u to enter so long as $\overline{q}_g \geq \frac{r}{b} + \frac{1}{G} + \frac{\Delta}{G}$.

If instead $\overline{q}_g < \frac{r}{b} + \frac{1}{G} + \frac{\Delta}{G}$, how can $\overline{q}$ be dismissed as an equilibrium? By showing that, in such a case, any g makes a loss at $\overline{q}$:

$$\pi_g(\overline{q}) = \frac{(r - b\Delta + bG\overline{q}_g)^2}{b(1+G)^2} - r\overline{q}_g < 0. \tag{11}$$

In fact, we can establish this inequality even when

$$\overline{q}_g = \frac{r}{b} + \frac{1}{G} + \frac{\Delta}{G}, \tag{12}$$

that is, when u makes zero profits if deviating to $\overline{q}_u = 1$. Making use of (12), our desired inequality (11) turns out to amount to $r(1+G)[G(1-\Delta) - \Delta - 1] + bG < 0$, hence to $(b\overline{q}_g - \frac{b\Delta}{G} - \frac{b}{G})(1+G)[G(1-\Delta) - \Delta - 1] + bG < 0$. Since the left-hand side is decreasing in $\overline{q}_g$, it suffices to prove this inequality for $\overline{q}_g = 2$, when it becomes

$$G^2 < (2G - \Delta - 1)(1+G)[\Delta + 1 + G(\Delta - 1)]. \tag{13}$$

Validity of (13) is immediate once it is recalled that $1 \leq \Delta < G$: then, on the right-hand side, $(2G - \Delta - 1)(1+G) > G^2$ and $\Delta + 1 + G(\Delta - 1) > 1$. ■

[13] For any given G, one requisite for maximal J is that $\overline{q}_j = 1$ for any $j : \overline{q}_j < \overline{q}_g$, in which case $J = D(r) + \Delta - G\overline{q}_g = n^* + \Delta - G\overline{q}_g$. This in its turn is maximal when $\overline{q}_g = 2$ and $\Delta = G - 1$ (given the constraint that $G > \Delta$), resulting in $J = n^* - 1 - G$. Therefore, maximal n equals $n^* - 1$.

[14] Note that, at $\overline{q}$, total equilibrium output is $\sum_{j:q_j<\overline{q}_l} \overline{q}_j + L\overline{q}_l + Gq_g < D(r)$, where $L = \#\{i : \overline{q}_i = \overline{q}_l\}$. Therefore, $\sum_{j:q_j<\overline{q}_l} \overline{q}_j + (L+G)\overline{q}_l \leq D(r) - 1$ given that $\overline{q}_l < q_g$ and capacities are integers. Consequently, total output at the new subgame equilibrium is $\sum_{j:q_j<\overline{q}_l} \overline{q}_j + 1 + (L+G)q_l \leq D(r)$ since $q_l \leq \overline{q}_l$.

B. Allowing for noninteger D(r)

B.1. An alternative "competitive" benchmark

As already seen, a long-run competitive equilibrium - based on the assumption of price-taking entrants - does not exist when $D(r) \notin \mathcal{F}_+$. In any such case, let us redefine the "competitive" capacity $\overline{Q}^*$ as the largest capacity consistent with nonnegative profits under market-clearing and the "competitive" price p^* as the corresponding market-clearing price: $\overline{Q}^* = (D(r) - 1, D(r)] \cap \mathcal{F}_+$ and $p^* = P(\overline{Q}^*)$. A justification for this terminology can be provided in terms of a two-stage capacity and quantity game with a competitive output market. In such a game, potential entrants choose capacities at stage 1, whereas, at stage 2, active firms choose quantities while taking the market price as given (its being set by an auctioneer equating demand and total supply). Note that, in this setting, while active firms are price takers, potential entrants do recognize how their capacity decisions are going to affect stage-2 market-clearing price. (See Dixon, 1985, where investment decisions by entrants are studied in a similar setting.) We refer to any subgame-perfect equilibrium outcome of such a game as a "long-run equilibrium with competitive pricing" (LRECP).

Let $\delta = D(r) - \overline{Q}^* < 1$. Note that $\overline{Q}^* \in (\frac{a-r-b}{b}, \frac{a-r}{b}]$ and $p^* = r + b\delta$. Now we are able to characterize any LRECP.

Proposition 4 *At an LRECP, the industry configuration is any $\overline{q}^*$, resulting in price p^*.*

Proof. At $\overline{q}^*$ the market-clearing price is p^*. Any potential entrant has made a best response: a deviation to $\overline{q}_i > 1$ by any $i \in \mathcal{A}^*$ or to $\overline{q}_u > 0$ by any $u \notin \mathcal{A}^*$ would yield losses at the new market-clearing price. In contrast, no $\overline{q} \notin \{\overline{q}^*\}$ can occur at an LRECP. At $\overline{q} : \overline{Q} \leq \overline{Q}^* - 1$, it pays any $u \notin \mathcal{A}$ to deviate to $\overline{q}_u = 1$.[15] At $\overline{q} : \overline{Q} > \overline{Q}^*$, $\pi_i^w(\overline{q}) < 0$ for any $i \in \mathcal{A}$. Thus we are left with $\overline{q} : n < \overline{Q} = \overline{Q}^*$. Then it always pays g to deviate to $\overline{q}_g - 1$. This is immediate when $D(r) \in \mathcal{F}^+$, since then $\pi_g^w(\overline{q}) = 0$. With $D(r) \notin \mathcal{F}^+$, it is still $\pi_g^w(\overline{q}_g - 1, \overline{q}_{-g}) > \pi_g^w(\overline{q})$. This can be seen using eq. (7) and the fact that $\overline{q}_g - 1$ is closer to $\overline{q}_g^\dagger = \frac{\overline{q}_g + \delta}{2}$ than $\overline{q}_g$ is: if $\overline{q}_g > 2$, then $\overline{q}_g^\dagger < \overline{q}_g - 1 < \overline{q}_g$; if $\overline{q}_g = 2$, then $\overline{q}_g^\dagger - (\overline{q}_g - 1) = 0.5\delta < \overline{q}_g - \overline{q}_g^\dagger = 1 - 0.5\delta$. ■

[15] If $D(r) \in \mathcal{F}_+$, then any $\overline{q} : \overline{Q} = \overline{Q}^* - 1$ is disposed of by our assumption that entry is strictly preferred to not entering.

To sum up, at each LRECP active firms have the minimum efficient size and total output is the largest one yielding nonnegative profits under market clearing.[16]

B.2. The capacity and price game

Now we turn to the capacity and price game. Recall that $\overline{Q}^* \in (\frac{a-r-b}{b}, \frac{a-r}{b}]$ and $p^* = r + b\delta \in [r, r+b)$; furthermore, $\widetilde{p}_i^* = \frac{r+b+b\Delta}{2} \in [\frac{r+b}{2}, \frac{r+2b}{2})$. In what follows, we will confine ourselves to capacity configurations inside $\overline{\mathcal{Q}}^* \cup \overline{\mathcal{Q}}^{**}$: in fact, configurations outside this set cannot occur at an equilibrium of the capacity and price game.[17]

Similarly to when $D(r) \in \mathcal{F}_+$, inequality $b \leq r$ guarantees the emergence of p^* at an equilibrium of the capacity and price game.

Lemma 6. *If $b \leq r$, then at an equilibrium of the capacity and price game the capacity configuration is any $\overline{q}^*$ and prices are set at p^* on the equilibrium path.*

Proof. A p.s.e. for the price subgame arises at $\overline{q}^*$ if and only if $\delta \geq \frac{b-r}{b}$, which certainly holds true when $b \leq r$. At $\overline{q}^*$ any firm has made a best capacity response. As to any $i \in \mathcal{A}^*$, if $\widetilde{p}_i^* \leq P(\overline{Q}^* + 1)$ the argument runs as with $D(r) \in \mathcal{F}^+$. If $\widetilde{p}_i^* > P(\overline{Q}^* + 1)$, then a m.s.e. for the price subgame will obtain if i deviates to $\overline{q}_i > 1$, resulting in $\pi_i(\overline{q}_i, \overline{q}_{-i}^*) = \pi_i^w(\widetilde{q}_i^*, \overline{q}_{-i}^*) - r(\overline{q}_i - \widetilde{q}_i^*)$. Even $\pi_i^w(\widetilde{q}_i^*, \overline{q}_{-i}^*) < \pi_i(\overline{q}^*)$: this can be checked using eq. (7) and the fact that $\overline{q}_i^\dagger < 1 = \overline{q}_i^*$. As to any $u \notin \mathcal{A}^*$, entering would lead to losses. Suppose u deviates to $\overline{q}_u = 1$ and a m.s.e. obtains for the price subgame (otherwise our case is trivial), which is so if and only if $\delta < \frac{2b-r}{b}$. Then u's profit reads $\pi_i(\overline{q}^{**}) = \widetilde{p}_i^{**}\widetilde{q}_i^{**} - r$, where $\widetilde{q}_i^{**} < 1$ and $\widetilde{p}_i^{**} = \frac{r+b\delta}{2} < r$ since $b \leq r$. ■

On the other hand, unlike with $D(r) \in \mathcal{F}_+$, inequality $b \leq r$ may not be necessary for the emergence of p^* at an equilibrium of the capacity and price game. To see why, note that, as $\delta \to 1$, $\widetilde{p}_i^* \to \frac{r+2b}{2}$ while $p^* \to r + b > \frac{r+2b}{2}$. Thus, $\widetilde{p}_i^* \leq p^*$ for δ sufficiently close to 1: at $\overline{q}^*$ the price subgame may have a p.s.e., even when $b > r$. With this insight, we can now address full equilibrium characterization.

Proposition 5 *At an equilibrium of the capacity and price game: (i) If $b \leq r$ or $b > r$ and $\frac{b-r}{b} \leq \delta < \frac{2\sqrt{br}-r}{b}$, then the capacity configuration*

[16] Noteworthy, this is how the long-run competitive equilibrium is actually defined in microeconomic textbooks such as Varian (1984, pp. 85-90).

[17] The proof of this claim is omitted, for brevity.

is any $\overline{q}^*$ and prices are set at p^* on the equilibrium path; (ii) if $b > r$ and $\delta < \min\{\frac{b-r}{b}, \frac{2\sqrt{br}-r}{b}\}$, then the capacity configuration is any $\overline{q}^*$ and a m.s.e. for the price subgame obtains on the equilibrium path; (iii) if $b > r$ and $\delta \geq \frac{2\sqrt{br}-r}{b}$, then the capacity configuration is any $\overline{q}^{**}$ and a m.s.e. obtains for the price subgame on the equilibrium path.

Proof. (i) A p.s.e. obtains at $\overline{q}^*$ given that $\frac{b-r}{b} \leq \delta$. At $\overline{q}^*$ any firm has made a best capacity response: in particular, if any $u \notin \mathcal{A}^*$ deviates to $\overline{q}_u = 1$ and a m.s.e. obtains for the price subgame, then u's expected revenue reads $\Pi_i(\overline{q}^{**}) = \widetilde{p}_i^{**}\widetilde{q}_i^{**} = \frac{(r+b\delta)^2}{4b}$: this is less than r because $\delta < \frac{2\sqrt{br}-r}{b}$.

(ii) A m.s.e. obtains at $\overline{q}^*$. It might easily be seen that best capacity responses have been made at stage 1.

(iii) Inequality $b > r$ guarantees that a m.s.e. obtains at $\overline{q}^{**}$; furthermore, $\pi_i(\overline{q}^{**}) \geq 0$ since $\delta \geq \frac{2\sqrt{br}-r}{b}$. Each firm has made a best capacity response. In particular, should any $u \notin \mathcal{A}^{**}$ deviate to $\overline{q}_u = 1$, then the industry configuration would become $\overline{q}^{***} : n^{***} = \overline{Q}^{***} \equiv \overline{Q}^* + 2$. Firm u's expected profit would be $\pi_i(\overline{q}^{***}) = \widetilde{p}_i^{***}\widetilde{q}_i^{***} - r < 0$ because $\widetilde{q}_i^{***} < 1$ and $\widetilde{p}_i^{***} = \frac{r+b\delta-b}{2} < r$. On the other hand, configurations $\overline{q}^*$ cannot arise at an equilibrium: any $u \notin \mathcal{A}^*$ would earn $\pi_i(\overline{q}^{**}) \geq 0$ by deviating to $\overline{q}_u = 1$. ■

B.3. The capacity and quantity game

Equilibria of the capacity and quantity game will be searched in the region $\{\overline{q} \mid n = \overline{Q}\}$ of the space of capacity configurations. By so doing we will actually discover *any* equilibrium so long as Lemma 5 extends to the case of $D(r) \notin \mathcal{F}_+$ (something we do not attempt to prove).

As in the capacity and price game, inequality $b \leq r$ guarantees the emergence of p^* at an equilibrium of the capacity and quantity game.

Lemma 7. *If $b \leq r$, then at an equilibrium of the capacity and quantity game the capacity configuration is any $\overline{q}^*$ and the firms produce their capacity on the equilibrium path, which results in market price p^*.*

Proof. Inequality $b \leq r$ is sufficient for a boundary solution to obtain for the quantity subgame at $\overline{q}^*$. Any firm has made a best capacity response. As to any $i \in \mathcal{A}^*$, let k be the integer such that $k - 1 < \widetilde{q}_i^* \leq k$. If deviating to $\overline{q}_i = k$, then, at the equilibrium of the new quantity subgame, $q_i = \widetilde{q}_i^*$ and $q_j = \overline{q}_j^* = 1$ for any $j \neq i \in \mathcal{A}^*$. Suppose first $k \geq 3$ so that $\widetilde{q}_i^* > 2$. Then losses are made because $Q = \sum_{j \neq i} \overline{q}_j^* + \widetilde{q}_i^* = \frac{a-r}{b} - \delta - 1 + \widetilde{q}_i^* > D(r)$ and hence $P(Q) < r$. Losses are also made if deviating to $\overline{q}_i \in \{2, ..., k-1\}$, in which case $q_i = \overline{q}_i$ and $q_j = 1$ for any $j \neq i \in \mathcal{A}^*$, again resulting in

$Q > D(r)$. With $k = 2$, deviating to $\overline{q}_i = 2$ leads to $q_i = \widetilde{q}_i^*$ and $q_j = 1$ for any $j \neq i \in \mathcal{A}^*$. This affords the deviant a profit of $\pi_i^w(\widetilde{q}_i^*, \overline{q}_{-i}^*) - r(\overline{q}_i - \widetilde{q}_i^*)$: and, since $\widetilde{q}_i^* > 1 > \overline{q}_i^\dagger$, even $\pi_i^w(\widetilde{q}_i^*, \overline{q}_{-i}^*) < \pi_i^w(\overline{q}^*)$. Turn now to any $u \notin \mathcal{A}^*$ and suppose it deviates to $\overline{q}_u = 1$. The quantity subgame has an internal equilibrium (the case of a boundary equilibrium is trivial) if $\delta < (2b - r)/b$: but then, as one can easily check, $Q = \frac{an^{**}}{b(n^{**}+1)} > \frac{a-r}{b}$, hence $P(Q) < r$. ∎

Again similarly to the capacity and price game, inequality $b \leq r$ may not be needed for the emergence of p^* at an equilibrium of the capacity and quantity game.

Proposition 6 *At an equilibrium of the capacity and quantity game: (i) If $b \leq r$ or $b > r$ and $\frac{b-r}{b} \leq \delta < \frac{a+2b-r-a\sqrt{b/r}}{b}$, then the capacity configuration is any $\overline{q}^*$ and the firms produce their capacity on the equilibrium path, resulting in market price p^*; (ii) if $b > r$ and $\delta \geq \frac{a+2b-r-a\sqrt{b/r}}{b}$, then the capacity configuration is any $\overline{q}^\S$ such that $n^\S = \overline{Q}^\S$ and $n^\S \leq -1 + \frac{a\sqrt{br}}{br} < n^\S + 1$, and the firms produce below capacity on the equilibrium path.*

Proof. (i) In view of Lemma 7, we only need to deal with the $b > r$ case As $\frac{b-r}{b} \leq \delta$, a boundary solution obtains at $\overline{q}^*$. Any firm has made a best capacity response. In particular, by deviating to $\overline{q}_u = 1$, any $u \notin \mathcal{A}^*$ would earn revenue $\Pi_i(\overline{q}^{**}) = \frac{a^2}{b(n^{**}+1)^2} = \frac{a^2 b}{(a+2b-r-b\delta)^2}$: this is less than r given that $\delta < \frac{a+2b-r-a\sqrt{b/r}}{b}$. This also allows us to dispose of any $\overline{q} : n = \overline{Q} > \overline{Q}^*$.

(ii) At $\overline{q}^\S$, $\Pi_i = \frac{a^2}{b(n^\S+1)^2} \geq r$ for each $i \in \mathcal{A}^\S$. Any firm has made a best capacity response. As to any $i \in \mathcal{A}^\S$, raising capacity would just raise costs; as to any $u \notin \mathcal{A}^\S$, entering would lead to losses. Configurations $\overline{q}$ such that $n = \overline{Q} > \overline{Q}^\S$ are immediately disposed of, since $\pi_i(\overline{q}) < 0$. ∎

B.4. Examples and discussion

Examples. 1. We begin illustrating the case where the "competitive" outcome obtains in either game. Let $a = 12$, $r = 1.2$, and $b = 1.25$, so that $D(r) = 8.64$, $\overline{Q}^* = 8$, and $p^* = 2$. Note that $\frac{b-r}{b} \leq \delta < \min\{\frac{2\sqrt{br}-r}{b}, \frac{a+2b-r-a\sqrt{b/r}}{b}\}$. Then, in either game, the equilibrium capacity configuration is any $\overline{q}^*$ and, on the equilibrium path: prices are set at p^* at the equilibrium of the price subgame; the firms produce their capacity at the equilibrium of the quantity subgame, which results in market price p^*.

2. We now illustrate the possibility of equilibrium total capacity being higher than $\overline{Q}^*$ at either game. Let $a = 12$, $r = 1.2$, and $b = 2.75$, so

that $D(r) = \frac{1080}{275}$, $\overline{Q}^* = 3$, $\delta = \frac{255}{275}$, and $p^* = 3.75$. Now, $b > r$ and $\delta \geq \frac{2\sqrt{br}-r}{b}$. Thus, at any equilibrium of the capacity and price game there are $n^{**} = 4$ firms, each with one unit of capacity: on the equilibrium path a m.s.e. is played for the price subgame, resulting in $\pi_i(\overline{q}^{**}) = \widetilde{p}_i^{**}\widetilde{q}_i^{**} - r = 1.875 \times \frac{1.875}{2.75} - 1.2 \cong .078$. As to the capacity and quantity game, since $\delta \geq \frac{a+2b-r-a\sqrt{b/r}}{b}$, at any equilibrium there are $n^{\S} = 5$ active firms, each with one unit of capacity. On the equilibrium path every active firm produces $\frac{a}{b(n^{\S}+1)} = \frac{24}{33}$, resulting in $P(Q^{\S}) = 2$ and $\pi_i(\overline{q}^{\S}) = 2 \times \frac{24}{33} - 1.2 \approx 0.2545$.

3. Finally, we illustrate the possibility of the "competitive" outcome emerging in the capacity and price game but not in the capacity and quantity game. Let $a = 13$, $r = 0.4$, and $b = 1$, so that $D(r) = 12.6$, $\overline{Q}^* = 12$, and $p^* = 1$. It is $\frac{b-r}{b} \leq \delta < \frac{2\sqrt{br}-r}{b}$, hence at any equilibrium of the capacity and price game there are $n = 12$ active firms, each one charging $p^* = 1$. On the other hand, $b > r$ and $\delta \geq \frac{a+2b-r-a\sqrt{b/r}}{b}$. Therefore, at any equilibrium of the capacity and quantity game there are $n^{\S} = 19$ active firms, each with one unit of capacity and producing $\frac{a}{b(n^{\S}+1)} = 0.65$; the market price is $P(Q^{\S}) = 0.65$ and $\pi_i(\overline{q}^{\S}) = 0.65^2 - 0.4 = 0.0225$. $\diamond$

The theoretical possibility illustrated by the last example above deserves further consideration. One requisite for the emergence of p^* at an equilibrium of the capacity and price game is that, at $\overline{q}^*$, it does not pay any $u \notin \mathcal{A}^*$ to enter. It might be so even if, at $\overline{q}^{**}$ - the configuration actually in place if u deviates to $\overline{q}_u = 1$ -, the price subgame had a m.s.e., provided $\widetilde{p}_i^{**}\widetilde{q}_i^{**} < r$. Turn now to the capacity and quantity game. Note that, at $\overline{q}^{**}$, equilibrium revenue is $\frac{a^2}{b(n^{**}+1)} > \widetilde{p}_i^{**}\widetilde{q}_i^{**}$: in fact, $\widetilde{p}_i^{**}\widetilde{q}_i^{**}$ is the revenue of the Stackelberg follower when each rival is supplying its unitary capacity, whereas $\frac{a^2}{b(n^{**}+1)}$ is the revenue of the Stackelberg follower when each rival is supplying $\frac{a}{b(n^{**}+1)} < 1$. Thus, it may be $\widetilde{p}_i^{**}\widetilde{q}_i^{**} < r < \frac{a^2}{b(n^{**}+1)}$: in such a case, $\overline{q}^*$ is an equilibrium of the capacity and price game but not of the capacity and quantity game because, in the latter, at $\overline{q}^*$ it pays any $u \notin \mathcal{A}^*$ to deviate to $\overline{q}_u = 1$.

C. A static capacity and quantity game

In terms of choice variables, any equilibrium outcome of the two-stage capacity and quantity game is a "capacity and output configuration", namely, a $z-$component vector of capacity and output pairs $((\overline{q}_1,\ q_1), ..., (\overline{q}_z, q_z))$.

It is easy to prove that any (subgame-perfect) equilibrium outcome of the two-stage capacity and quantity game is an equilibrium of the static game. Consider any firm i such that $(\overline{q}_i, q_i) > (0,0)$. Over the range $[0, \overline{q}_i]$, q_i is clearly a best response to the rivals' output. Also, it does not pay any firm i (no matter whether $(\overline{q}_i, q_i) > (0,0)$ or $(\overline{q}_i, q_i) = (0,0)$) to deviate to a higher capacity and then eventually adjust output: such a move is unprofitable in the two-stage setting, where the rivals' output at the equilibrium of the quantity subgame is nonincreasing in i's capacity; therefore, since outputs are strategic substitutes, prospects are even worse in the static game, where the rivals' output is taken as given.

On the other hand, the static game may have Nash equilibria that are not (subgame-perfect) equilibria of the two-stage game. Let $b > a^2 b/(a + 2b - r)^2 \geq r$, as in part (ii.b) of Proposition 2. We already know that any $\overline{q}^{\S}$ coupled with $q_i = \frac{a}{b(n^{\S}+1)}$ for each $i \in \mathcal{A}^{\S}$ is an equilibrium of the static game. Further equilibria are identified as follows. Consider any capacity and output configuration with n active firms - where $n^* \leq n < n^{\S}$ - and $q_i = \frac{a}{b(n+1)} < \overline{q}_i = 1$ for any $i \in \mathcal{A}$. Clearly any $i \in \mathcal{A}$ has made a best response. Therefore, for any such configuration to be an equilibrium it must be that any $u \notin \mathcal{A}$ has also made a best response. Note that the best deviation u can make is the capacity-output pair $(\overline{q}_u = 1, q_u = \frac{a}{2b(n+1)})$, where $\frac{a}{2b(n+1)} < 1$ is u's best output response (with $\overline{q}_u > 0$) to the rivals' total output. This results in $\Pi_u = \frac{a^2}{4b(n+1)^2}$, which is less than r if and only if $n > -1 + \frac{a\sqrt{br}}{2br}$. Thus we have this result: if $b > \frac{a^2 b}{(a+2b-r)^2} \geq r$, then any capacity and output configuration with $n \in (\max\{n^*, -1 + \frac{a\sqrt{br}}{2br}\}, -1 + \frac{a\sqrt{br}}{br}]$ active firms and $q_i = \frac{a}{b(n+1)} < \overline{q}_i = 1$ for any $i \in \mathcal{A}$ constitutes an equilibrium. Applying this result to Example 3 on p. 11, it can be checked that any capacity and output configuration with $n \in (8, 11]$ active firms, each with $\overline{q}_i = 1$ and producing $q_i = \frac{17}{2(n+1)}$ is an equilibrium of the capacity and quantity game.